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ANALYSIS OF ELASTIC WAVE DYNAMICS IN WALLS OF A SPHERICAL EXPLOSION CHAMBER

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UDC 533 + 539

The wave motion is investigated numerically, and the magnitude of the elastic stresses is estimated in the walls of a spherical explosion chamber.

The theoretical computational model of gasdynamic and mechanical processes proceeding in a spherical explosion chamber was examined in detail in [1]. The proposed model permitted computation of the wave motion parameters within the chamber and estimation of the fraction of energy transmitted to its walls. A detailed comparison between the numerical results obtained and certain experimental—computational data [2] showed good agreement. The investigation executed in [1] permitted the conclusion that the model assures a more rigorous analysis of the phenomena under consideration as compared with the assumptions often used in the literature about the constancy of the pressure on the chamber walls or the possibility of approximating it by the simplest analytic dependences [3, 4].

The present investigation supplements [1] in the numerical study of the dynamics of elastic waves being generated in chamber walls subjected to periodic pulsed impacting loads for the case of finitely or infinitely thick walls, which is of interest for the solution of a broad class of practical problems [2-5].

Within the framework of the model proposed in [1], wave processes in the walls of a spherical explosion chamber of radius ~ 2 m filled with air of the density $\rho_0 = 1.293 \cdot 10^{-3}$ g/cm^3 at a pressure $P_0 = 1$ atm at whose center is an energy-liberating source of 3 cm radius and $E_0 = 7.106 \cdot 10^9$ J energy are considered in this paper. Aluminum ($\rho_1 = 2.7$ g/cm^3) was selected as chamber wall material.

Scientific-Research Institute of Applied Physical Problems at the V. I. Lenin Belorussian State University, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 5, pp. 838-843, November, 1984. Original article submitted April 27, 1983.

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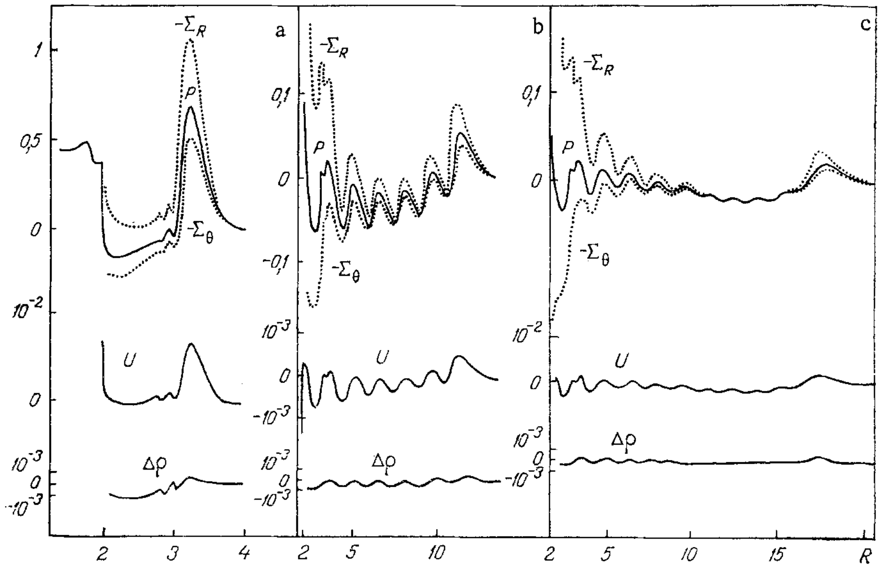

Fig. 1. Distribution of the pressure P (kbar), principal stresses $-\Sigma_R$ and $-\Sigma_\theta$ (kbar), velocity U (km/sec), and relative change in density $\Delta\rho$ at the chamber walls along the radius R (m) for different times t (msec): a) 0.299; b) 1.567; c) 2.57. Infinite chamber-wall thickness version.

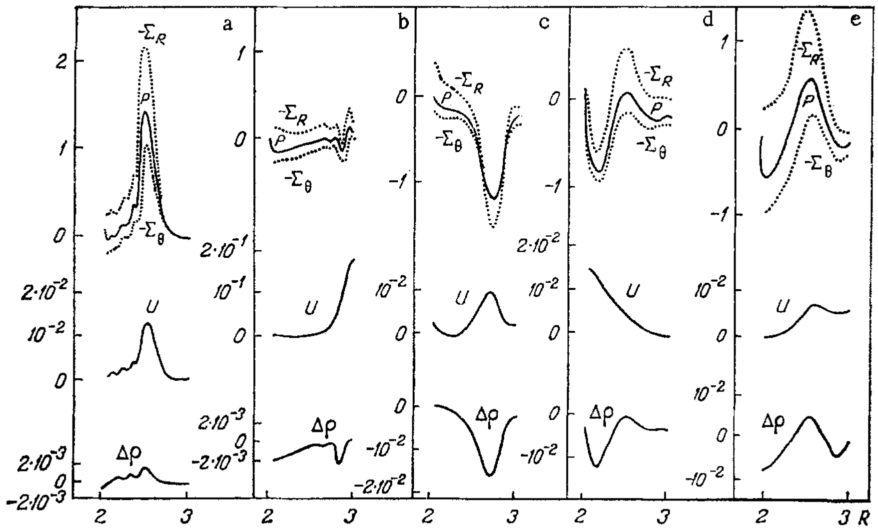

Fig. 2. Distribution of the pressure P (kbar), principal stresses $-\Sigma_R$ and $-\Sigma_\theta$ (kbar), velocity U (km/sec), and relative change in density $\Delta\rho$ at the chamber wall over the coordinate R (m) for different times t (msec): a) 0.1738; b) 0.2512; c) 0.3021; d) 0.3982; 3) 0.5012. Finite wall thickness version.

According to [1], the system of continuum mechanics equations

$$\dot{R}=U,\ \dot{U}=V_1\left[\frac{R}{M}\right]^2\frac{\partial\Sigma_R}{\partial M}+2\frac{\Sigma_R-\Sigma_\theta}{R},$$

$$V=V_1\left[\frac{R}{M}\right]^2\frac{\partial R}{\partial M},\ \dot{E}=V(s_1\dot{\varepsilon}_1+2s_2\dot{\varepsilon}_2)-(P+q)\dot{V},$$

where $\Sigma_R=-(P+q)+s_1;\ \Sigma_\theta=-(P+q)+s_2;\ \dot{s}_1=2\mu_1\left(\dot{\varepsilon}_1-\frac{1}{3}\frac{\dot{V}}{V}\right);\ \dot{s}_2=2\mu_1\left(\dot{\varepsilon}_2-\frac{1}{3}\frac{\dot{V}}{V}\right);\ \dot{\varepsilon}_1=\frac{\partial U}{\partial R};$ $\dot{\varepsilon}_2=\frac{U}{R}$, was used for a numerical computation of the processes both within the chamber and

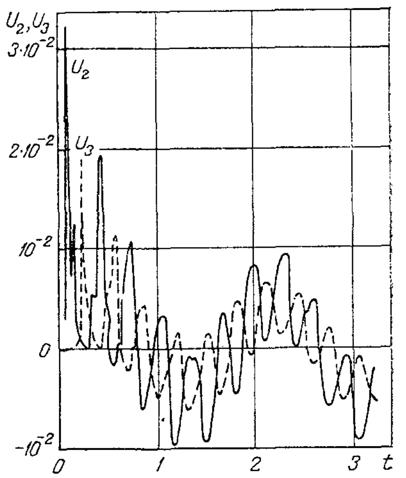

Fig. 3

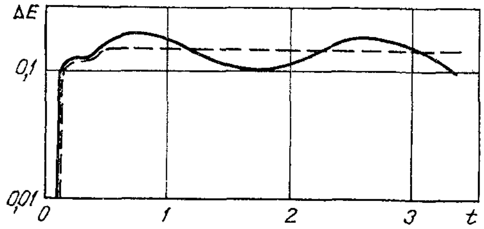

Fig. 4

Fig. 3. Velocities (km/sec) of the internal U_2 and external U_3 chamber wall boundaries as a function of the time t (msec).

Fig. 4. Dependence of the energy ΔE (%) transmitted to the chamber wall on the time t (msec). Dashed line corresponds to the case of unlimited chamber wall thickness, and solid line to a finite thickness wall.

on its walls. The dots above the quantities in the equations denote the derivative with respect to time along the trajectory of the medium particles.

The Mises condition $s_1^2 + s_2^2 + s_3^2 \leq 2/3Y_0^2$, $s_3 = -(s_1 + s_2)$, was used to correct the stress deviator components during passage to the plastic flow stage of the medium; the values of the yield point and shear modulus for aluminum are respectively [6, 9] $Y_0 = 2.976$ kbar and $\mu_1 = 248$ kbar.

Therefore, the description of the wave processes in chamber walls used in this paper takes account of the elastic, elastic–plastic, and hydrodynamic stages of the motion of the medium and is therefore of sufficiently general nature.

The computation of the thermodynamic parameters in the chamber walls was performed by means of the Tillotson equations which reflect the properties of the medium well under pulsed impact loads [7-9]:

$$P = [a + b/(E/c\eta^2 + 1)]\, E\rho + A\mu + B\mu^2,\ \rho \geqslant \rho_1,$$

$$P = aE\rho + \left[bE\rho/(E/c\eta^2 + 1) + A\mu \exp\left(-\beta\left(\frac{1}{\eta} - 1\right)\right)\right] \exp\left(-\alpha\left(\frac{1}{\eta} - 1\right)^2\right),\ \rho < \rho_1,$$

where $\mu = \eta - 1$; $\eta = \rho/\rho_1$.

The following values of the constants in the equations of state for aluminum were taken for the versions of the problem under investigation [9]: $\rho_1 = 2.7$ g/cm^3; $A = 7.5 \cdot 10^{11}$ dyn/cm^2; $B = 6.5 \cdot 10^{11}$ dyn/cm^2; $a = 0.5$; $b = 1.63$; $\alpha = 5.0$; $\beta = 5.0$; $c = 5 \cdot 10^{11}$ erg/g.

As follows from [1], the process of explosive loading of chamber walls is a periodic pulse in nature, and is determined by the dynamics of shockwave motion. A definite vibration frequency ∿4 kHz is built up in the chamber with time; here the amplitude of the changes in pressure, density, and temperature on the walls drops sharply after several of the first pulses, and later remains practically constant [1].

A field of elastic stresses is generated in the walls as a result of the periodic wave motion within the chamber. A train of elastic compression–tension waves (Fig. 1), of oscillatory structure due to the pulsed weakly-damped nature of the shock processes in the chamber [1], is here propagated in the medium. The elastic pulses being generated in the chamber walls with the lapse of time lose their energy and are smoothed out so that in practice only the first of them (Fig. 1) with an excess pressure magnitude of ∿20 bar (t = 2.57 msec) at the front is clearly looked over after several msec. The spatial elastic-wave deformation observed in the graphs and the sharp drop in their amplitude are explained by the intensive energy absorption in the area of elastic tension of the medium, which has mainly positive values of the stresses that occur after the first pulse (Fig. 1).

The results presented in Fig. 1 refer to the case of an unlimited dense medium surrounding the chamber. The process of wave motion development and elastic stress field formation in a chamber wall with a 1 m thickness is represented in Fig. 2. In contrast to the unlimited, dense medium surrounding the chamber, taking account of the finite wall dimensions results in complication of the physical pattern of the phenomena being studied.

The elastic waves that occur in the chamber wall under the effect of shock pulses are reflected from the media interfaces, change the motion direction, are mutually amplified or attenuated, consequently resulting in a sufficiently complicated pattern of the stress distribution in the medium (Fig. 2) due to the many components. As is seen from Fig. 2, for a continuously varying elastic wave field the magnitude of the stresses occurring in the chamber wall does not exceed the strength limit of 1-3 kbar for aluminum [10]. The relative change in density because of the elastic compression–tension of the medium is here not more than ∿1-2%, which is considerably greater than the relative change in density in the case of a wall of unlimited thickness (tenths and hundredths of a percent (Fig. 1)) and is due to chamber wall tension as a whole.

Results of numerical computations permitted it to be established that the action of shock pulses on a finite-thickness wall results in the origination of natural wall vibrations with a period of ∿2 msec in the version of the problem under consideration (Fig. 3). In particular, the shape of the curve for the energy absorption function in the chamber wall (Fig. 4) is explained by this effect. In the finite wall thickness case, this function oscillates slowly around the quantity ∿0.15%, corresponding to the version of the unlimited medium [1] surrounding the chamber.

In conclusion, we note that an analysis of the numerical results obtained in this paper and their comparison with the data in [2] permits making the deduction that elastic waves originating in explosion chamber walls under the effect of pulsed impact loads, as was assumed in [1], have practically no influence on the pattern of the gasdynamic motion within the chamber.

NOTATION

ρ, density; E, specific internal energy; ΔE, fraction of the energy radiation through the wall by the chamber; P, pressure; q, pseudoviscosity; V, specific volume; Σ_R, Σ_θ, radial and tangential stress; s_1, s_2, s_3, stress deviator components; ε_1, ε_2, ε_3, strain vector components; P_0, ρ_0, air pressure and density; ρ_1, density of the wall material; $\Delta\rho$, relative change in density in the chamber walls; E_0, source energy; μ_1, shear modulus; Y_0, yield point; A, B, a, b, c, α, β, constants in the Tillotson equation of state; R, radial coordinate; t, time; M, mass (Lagrange) coordinate; U, velocity; U_2, U_3, velocities of the inner and outer chamber wall boundaries.

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EFFECT OF MEMORY ON DISSIPATIVE STRUCTURES FORMING IN DISTRIBUTED KINETIC SYSTEMS

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UDC 532.59:536.7:541.121

Integrodifferential equations which include memory effects are proposed for describing the formation of dissipative structures in distributed kinetic systems.

Quite thorough studies have been made in recent years concerning dissipative structures in distributed kinetic systems describable by parabolic equations of transfer [1-5], these equations being derived from the conditions of balance and from phenomenological laws which express instantaneous and local relations between thermodynamic fluxes and forces on the assumption that local equilibrium prevails in every small element of the medium. The local state of the medium is, moreover, completely described by an equation which does not contain any gradients. In most models the kinetic transfer coefficients are assumed to be constant [1-3]. Equations of the parabolic kind with constant transfer coefficients admit solutions (not physically realistic) which yield infinitely large fluxes at time zero [6-8]. Despite these singularities in the solutions, the latter rather accurately describe experimental data obtained in studies of structurization during low-intensity transient processes. Singularities in the solutions to parabolic equations cause difficulties of theoretical nature, however, in description of experimental data obtained about dissipative structures in distributed active systems during fast nonequilibrium processes. In such processes the gradients are large and dispersion effects become significant so that it becomes necessary to include nonlocality and memory effects in the relations between thermodynamic fluxes and forces. It is then incorrect to describe the formation of structures with parabolic equations derived in accordance with conventional nonequilibrium thermodynamics, and equations of far-from-equilibrium thermodynamics are required instead.

Methods of nonlinear thermomechanics of continuous media yield the equations

$$\begin{aligned}
&\alpha_n(0)\frac{\partial^2 z_n(x,\ t)}{\partial t^2}+\beta_n(0)\frac{\partial z_n(x,\ t)}{\partial t}+\int\limits_0^\infty \beta_n'(\theta)\frac{\partial z_n(x,\ t-\theta)}{\partial t}\,d\theta=\\
&=k_n(0)\nabla^2 z_n(x,\ t)+\int\limits_0^\infty k_n'(\theta)\nabla^2 z_n(x,\ t-\theta)\,d\theta+\\
&+\sum_r \nu_{nr}\dot{W}_r'+\Phi(x,\ t),\\
&C_V\frac{\partial^2 T(x,\ t)}{\partial t^2}+\beta(0)\frac{\partial T(x,\ t)}{\partial t}+\int\limits_0^\infty \beta'(\theta)\frac{\partial T(x,\ t-\theta)}{\partial t}\,d\theta=\\
&=k(0)\nabla^2 T(x,\ t)+\int\limits_0^\infty k'(\theta)\nabla^2 T(x,\ t-\theta)\,d\theta=
\end{aligned} \tag{1}$$

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